Toughness of pseudorandom graphs

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The toughness $t(G)$ of a connected graph G is defined as $t(G) = \min\{\frac{|S|}{c(G-S)}\}$, in which the minimum is taken over all proper subset $S \subset V(G)$ such that $c(G-S) > 1$, where $c(G-S)$ denotes the number of components of $G - S$. Graph toughness was introduced by Chvátal in 1973 and is closely related to many graph properties, including Hamiltonicity, pancyclicity, factors, and spanning trees, etc.

A d-regular graph on n vertices with the second largest absolute eigenvalue at most λ is called an (n, d, λ)-graph. It is well known that an (n, d, λ)-graph for which $\lambda = \Theta(\sqrt{d})$ is a very good pseudorandom graph, behaving, in many aspects, like a truly random graph $G(n, p)$. For any connected d-regular graph G, it has been shown by Alon that $t(G) > \frac{1}{3}(\frac{d^2}{d\lambda+\lambda^2} - 1)$, through which, Alon was able to show that for every t and g there are t-tough graphs of girth strictly greater than g, and thus disproved in a strong sense a conjecture of Chvátal on pancyclicity. Brouwer independently discovered a better bound $t(G) > \frac{d}{\lambda} - 2$, and he also conjectured that the lower bound can be improved to $t(G) \geq \frac{d}{\lambda} - 1$. We filled the small gap and confirmed this 25-year-old conjecture.