

ON PARADOXICAL EXAMPLES OF REAL FUNCTIONS

CHENG-HAN PAN

ABSTRACT. This talk is a summary of the study of paradoxical real functions, supervised by Professor Krzysztof Chris Ciesielski, based on four publish articles. The material will be presented in two independent parts, where one considers differentiability and the other considers generalized continuity. Firstly, we will first introduce a relatively simple construction of a *differentiable monster*—a nowhere-monotone function that is differentiable everywhere—and discuss it in sense of Jordan decomposition, function extension, and Baire category. Secondly, we will construct a *Sierpiński-Zygmund functions* with transfinite induction. Adding an additional set-theoretical axiom, we will impose intermediate value property and list examples of *Sierpiński-Zygmund functions* in different *Darboux-like* classes.

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DEPARTMENT OF MATHEMATICS,
WEST VIRGINIA UNIVERSITY, MORGANTOWN,
WV 26506-6310, USA.
Email address: `chpan@mix.wvu.edu`

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