The 2024 WVU CONFERENCE ON
DISCRETE MATHEMATICS AND APPLICATIONS

April 19-21, 2024

School of Mathematical and Data Sciences
West Virginia University

To honor the retirements of Professors
John Goldwasser, Hong-Jian Lai, and Cun-Quan Zhang

Schedule of the conference

Reception, 7:00 -9:00 PM, Friday, April 19, 202 Hodges Hall

Talks on Saturday, April 20, Hodges Hall

Social Event, Sunday, April 21

Saturday Morning, April 20, 2024

	Room 210	Room 214	Room 220
8:00-8:30	Registration: Lobby Outside of 202 Hodges Hall		
8:30-9:00	Welcoming Remarks Dr. Duncan Lorimer, Associate Dean for Research, Eberly College of Arts and Sciences Room: 202 Hodges Hall		
9:00-10:00	Alternating Sign Matrices and Hypermatrices Richard Brualdi University of Wisconsin-Madison		
	Room 210	Room 214	Room 220
10:00-10:20	Ye Chen Some problems on r-hued colorings	Mingquan Zhan Pancyclicity of line graphs with diameter two	Pawel Pralat The Erdös- Gyárfás function $f(n,4,5) = 5/6\, n + o(n)$ --- so Gyárfás was right
10:20-10:40	Murong Xu Some problems on r-hued colorings	Sulin Song On Hamiltonian Properties of $K_{\{1,r\}}$-free Split Graphs	Xujun Liu Every subcubic graph is packing (1,1,2,2,3)-colorable
10:40-11:00	Lucian Mazza Unique Group Coloring of Graphs	Ju Zhou Unique Efficient Dominating Sets	Zi-Xia Song On tight (k,l)-stable graphs
11:00-11:20	**Coffee Break**		
11:20-12:20	A Solution to the Total Chromatic Number Conjecture Anthony Hilton University of Reading Cycle Covers of Graphs Dong Ye Middle Tennessee State University		
12:20-2:00	**Lunch Break**		

Saturday Afternoon, April 20, 2024

	Room 210	Room 214	Room 220
2:00-2:20	Gexin Yu On the $(1^2,2^4)$-packing edge-coloring of subcubic graphs	Yehong Shao A Survey on the Index Problem of Iterated Line Graphs	Cheng Han Pan Continued fractions, a-Fibonacci numbers, and the middle b-noise
2:20-2:40	Maya Tennant Cup Stacking in Graphs	Michael Mays Casting light on integer partitions	Jianbing Liu The structure of 2-matching connected graphs
2:40-3:00	Chong Li Integer flows on triangularly connected signed graphs	Lin Tian Removing long cycles in highly connected graphs with degree constraints	Elaine Eschen Bipartite completion of colored graphs avoiding chordless cycles of given lengths
3:00-3:20	**Coffee Break**		
3:20-4:20	Paths, Cycles, and Edge Coloring Guantao Chen Georgia State University		
4:20-6:00	Closing Ceremony 202 Hodges Hall		
6:45	Dinner: Kome Buffet		

List of speakers at the closing ceremony

(chaired by Rong Luo)

1. Richard A. Brualdi (University of Wisconsin-Madison)
2. Guantao Chen (Georgia State University)
3. Ye Chen (Northern Arizona University)
4. Chris Ciesielski (West Virginia University)
5. Harvey Diamond (West Virginia University)
6. Elaine Eschen (West Virginia University)
7. Michael Ferencak (University of Pittsburgh at Johnstown)
8. Chip Klostermeyer (University of North Florida)
9. Xuechao Li (University of Georgia)
10. Michael Mays (West Virginia University, Emeritus)
11. Paweł Prałat (Toronto Metropolitan University)
12. Zi-Xia Song (University of Central Florida)
13. Xiaoqiang Wang (Neumora Therapeutics)
14. Murong Xu (The University of Scranton)
15. Gexin Yu (William & Mary)
16. Mingquan Zhan (Millersville University)
17. Xiankun Zhang (OpenText)
18. Ju Zhou (Kutztown University)
19.

Abstracts

Contents

Alternating Sign Matrices and Hypermatrices

Richard A. Brualdi, *University of Wisconsin-Madison*
Saturday, April 20, 9:00-10:00, 202 Hodges Hall

From the perspective of combinatorial matrix theory, alternating sign matrices (ASMs) can be considered as generalizations of permutation matrices. We shall give a general introduction to this topic including some recent joint work with Geir Dahl and Michael Schroeder. A generalization to 3-dimensions is also considered. Latin squares can be regarded as 3-dimensional permutation matrices which leads to alternating sign hypermatrices, including the notion of orthogonality.

Paths, Cycles, and Edge-Coloring

Guantao Chen, *Georgia State University*
Saturday, April 20, 3:20-4:20, 202 Hodges Hall

In this talk, we cover several different topics in structural graph theory with the hope to reveal their connection. We will start a sufficient degree condition for a k-connected claw-free graph to have a hamiltonian cycle and the inserting vertices method, several problems on partitioning edges of graphs into cycles and/or paths, and their connection to edge coloring of multigraphs involving some density properties.

Some problems on r-hued colorings

Ye Chen, *Northern Arizona University*
Saturday, April 20, 10:00-10:20, 210 Hodges Hall

Let k, r, d be positive integers. A (k, r)-coloring of a graph G is a proper k-vertex coloring of G such that the neighbors of each vertex of degree d will receive at least $\min(d, r)$ different colors. The r-hued chromatic number is the smallest integer k for which a graph G has a (k, r)-coloring. In this talk, I will introduce the recent developments on the studies related to this r-hued coloring and discuss some of the open problems on it.

Joint work with Murong Xu, Suohai Fan, and Hong-Jian Lai.

Integer flows on triangularly connected signed graphs

Chong Li, *Southern Arkansas University*
Saturday, April 28, 2:40 - 3:00, 210 Hodges Hall

A triangle-path in a graph G is a sequence of distinct triangles $T_1, T_2, \ldots, T_m$ in G such that for any i, j with $1 \le i < j \le m$, $|E(T_i) \cap E(T_{i+1})| = 1$ and $E(T_i) \cap E(T_j) = \emptyset$ if $j > i + 1$. A connected graph G is triangularly connected if for any two nonparallel edges e and e' there is a triangle-path $T_1 T_2 \cdots T_m$ such that $e \in E(T_1)$ and $e' \in E(T_m)$. For ordinary graphs, Fan *et al.* characterize all triangularly connected graphs that admit nowhere-zero 3-flows or 4-flows. Corollaries of this result include integer flow of some families of ordinary graphs, such as, locally connected graphs due to Lai and some types of products of graphs due to Imrich et al. In this paper, Fan's result for triangularly connected graphs is

further extended to signed graphs. We proved that a flow-admissible triangularly connected signed graph admits a nowhere-zero 4-flow if and only if it is not the wheel W_5 associated with a specific signature. Moreover, this result is sharp since there are infinitely many unbalanced triangularly connected signed graphs admitting a nowhere-zero 4-flow but no 3-flow.

Bipartite completion of colored graphs avoiding chordless cycles of given lengths

Elaine Eschen, *West Virginia University*
Saturday, April 20, 2:40-3:00, 220 Hodges Hall

We consider a well-known restriction of the graph sandwich problem: Given a graph G with a proper vertex coloring, determine if there is a completion of G (formed by adding edges while maintaining the proper coloring) that has property P. We study completions that are bipartite graphs without induced cycles of prescribed lengths. It is known that deciding whether there is a chordal bipartite completion is NP-complete when the input graph is colored with an arbitrary number of colors. We consider the case when the input graph is 3-colored and show that the problem of deciding whether there is a bipartite completion that avoids induced cycles $C_6, \ldots, C_{2p}$, for fixed $p \geq 3$, is NP-complete. We characterize chordal bipartite completions of 3-colored graphs, and based on this, show that deciding whether a 3-colored graph can be completed to be chordal bipartite is solvable in $O(m + n\alpha(n))$ time. When a chordal bipartite completion exists, a size-n representation of a completion can be constructed within the same time bound. It follows from our results that for every fixed $k \geq 3$, and for every fixed $p \geq 3$, deciding whether a k-colored graph can be completed to be bipartite and $(C_6, \ldots, C_{2p})$-free is NP-complete. Also, the corresponding graph sandwich problems are hard.

This is joint work with R Sritharan from University of Dayton.

A Solution to the Total chromatic Number Conjecture

Anthony Hilton, *University of Reading*
Saturday, April 20, 11:00-12:00, 202 Hodges Hall

The total chromatic number $\chi_T(G)$ of a graph G is the smallest number of colours needed to colour the vertices and edges of G so that

1. no colour is used on two adjacent vertices,

2. no colour is used on two edges which have a common vertex,

3. no colour is used on a vertex v and an edge incident with v.

In 1964 and 1965 Behzad [1] and Vizing [2] independently conjectured that, for a simple graph G,

$$\chi_T \leq \Delta(G) + 2.$$

In this paper we prove this conjecture.

We actually prove a generalisation of this conjecture. Suppose that to each vertex and each edge there is assigned a paintbox (or list) so that if each paintbox contains $c_T(G)$ coloured paints, then G has a total

colouring (i.e. each edge and each vertex gets a colour which is selected from the paintbox for that edge or for that vertex). If all the paintboxes have the same size then the least size for which this is always possible whatever the choice of paintboxes is called the total choice number $c_T(G)$ of G. We show that, for a simple graph G, $c_T(G) \leq \Delta(G) + 2$. Clearly $\chi_T(G) \leq c_T(G)$.

[1] M. Behzad. Graphs and their chromatic numbers. In Ph.D. Thesis, Michigan State University, 1965.

[2] V.G.Vizing. On an estimate of the chromatic class of a p-graph. In Metody, Diskret. Analiz., Vol. 3, pages 9-17 (in Russian). 1964.

The structure of 2-connected graphs

Jianbing Liu, *University of Hartford*
Saturday, April 20, 2:20 - 2:40, 220 Hodges Hall

For an integer k, a graph G is *k-matching connected* if $G - V(M)$ is a connected nontrivial graph for each matching M with $|M| \leq k-1$, where $V(M)$ is the set of vertices covered by M. We show that a graph is 2-matching connected if and only if either $G \in \{C_4, K_4\}$ or it can be obtained from a C_4 by repeatedly applying three graph operations. In addition, for a k-matching-connected graph G, an edge $e \notin E(G)$ is said to be *addible* if $G + e$ is still k-matching-connected. We prove that every 2-matching-connected graph G has no addible edge if and only if G is a cycle of length at least 4. We also show that every 3-matching-connected graph G with $\delta(G) \geq 5$ has an addible edge, and some examples are given to show that $\delta(G) = 5$ is the best possible lower bound. Joint work with Hengzhe Li, Menghan Ma and et al.

Every subcubic graph is packing $(1, 1, 2, 2, 3)$-colorable

Xujun Liu, *Xi'an Jiaotong-Liverpool University*
Saturday, April 20, 10:20-10:40, 220 Hodges Hall

For a sequence $(s_1, \ldots, s_k)$ of non-decreasing integers, a packing S-coloring of a graph G is a partition of its vertex set $V(G)$ into $V_1, \ldots, V_k$ such that for every pair of distinct vertices $u, v \in V_i$ the distance between u and v is at least $s_i + 1$, where $1 \leq i \leq k$. The packing chromatic number, $\chi_p(G)$, of a graph G is defined to be the smallest integer k such that G has a packing $(1, 2, \ldots, k)$-coloring. Gastineau and Togni asked an open question "Is it true that the 1-subdivision $(D(G))$ of any subcubic graph G has packing chromatic number at most 5?" and later Brešar, Klavžar, Rall, and Wash conjectured that it is true.

In this paper, we prove that every subcubic graph has a packing $(1, 1, 2, 2, 3)$-coloring and it is sharp due to the existence of subcubic graphs that are not packing $(1, 1, 2, 2)$-colorable. As a corollary of our result, $\chi_p(D(G)) \leq 6$ for every subcubic graph G, improving a previous bound (8) due to Balogh, Kostochka, and Liu in 2019, and we are now just one step away from fully solving the conjecture. Joint work with Xin Zhang and Yanting Zhang.

Casting light on integer partitions

Michael Mays, *West Virginia University*
Saturday, April 20, 2:20-2:40, 214 Hodges Hall

Integer partitions of n are viewed as bargraphs (i.e., Young diagrams rotated anticlockwise by 90 degrees) in which the ith part of the partition x_i is given by the ith column of the bargraph with x_i cells. The sun is at infinity in the north west of our two dimensional model and each cell may or may not be lit depending on whether it stands in the shadow cast by another cell to its left. The partition being ordered either as weakly increasing or decreasing yields different generating functions and we consider both cases. We study the number of lit cells in partitions of n in their bargraph representation. In the weakly increasing case the generating function is straightforward, but in the weakly decreasing case we define triangular q-binomial coefficients which are analogous to standard q-binomial coefficients and we obtain a formula for these. We also study lit nodes in rotated Ferrers diagrams.

Unique Group Coloring of Graphs

Lucian Mazza, *Oakland University*
Saturday, April 20, 10:40-11:00, 210 Hodges Hall

Group Coloring is a "non-homogeneous" generalization of vertex coloring, where vertices are colored with group elements in such a way that adjacent vertices do not differ by a value assigned to the edge they share. This talk will discuss what it means for a group coloring to be unique, as an extension of the previously well-studied unique vertex coloring problem, and present some properties of graphs that can be uniquely colored under some group assignment.

CONTINUED FRACTIONS, a-FIBONACCI NUMBERS, AND THE MIDDLE b-NOISE

Cheng-Han Pan, *Western New England University*
Saturday, April 20, 2:00-2:20, 220 Hodges Hall

This talk is based on our students participation in solving problem 1385 in the Pi Mu Epsilon Journal. We show a closed-form expression of the continued fraction $[1, 1, ..., 1, 3, 1, 1, ..., 1]$ and a generalization to $[a, a, ..., a, b, a, a, ..., a]$ with a-Fibonacci numbers. In addition, we discuss how much the middle b-noise affects the continued fractions with all as.

The Erdős-Gyárfás function $f(n, 4, 5) = \frac{5}{6}n + o(n)$ — so Gyárfás was right

Paweł Prałat, *Toronto Metropolitan University*
Saturday, April 20, 10:00-10:20, 220 Hodges Hall

A $(4, 5)$-coloring of K_n is an edge-coloring of K_n where every 4-clique spans at least five colors. We show that there exist $(4, 5)$-colorings of K_n using $\frac{5}{6}n + o(n)$ colors. This settles a disagreement between Erdős and Gyárfás reported in their 1997 paper. Our construction uses a randomized process which we analyze using the so-called differential equation method to establish dynamic concentration. In particular,

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our coloring process uses random triangle removal, a process first introduced by Bollobás and Erdős, and analyzed by Bohman, Frieze and Lubetzky.
(This is a joint work with Bennett, Cushman, and Dudek.)

A Survey on the Index Problem of Iterated Line Graphs

Yehong Shao, *Ohio University Southern*
Saturday, April 20, 2:00-2:20, 214 Hodges Hall

The *line graph* of a graph G, denoted by $L(G)$ or $L^1(G)$, is a simple graph with $E(G)$ being its vertex set, where two vertices in $L(G)$ are adjacent if and only if the corresponding two edges in G have a common vertex. For any integer $n > 0$, the iterated line graphs are recursively defined by $L^n(G) = L(L^{n-1}(G))$ with $L^0(G) = G$. The concept of the Hamiltonian index of a graph G was first introduced by Chartrand as the least nonnegative integer k such that $L^k(G)$ is Hamiltonian. In 1983, Clark and Wormald extended this idea and introduced the Hamiltonian-like indices. Since then there has been a tremendous amount of papers on the indices. We provide a survey on these results and the main techniques applied to prove them.
Joint work with Liming Xiong and Hong-Jian Lai.

On Hamiltonian Properties of $K_{1,r}$-free Split Graphs

Sulin Song, *West Texas A&M University*
Saturday, April 20, 10:20 - 10:40, 214 Hodges Hall

Let $r \geq 3$ be an integer. A graph G is $K_{1,r}$-**free** if G does not have an induced subgraph isomorphic to $K_{1,r}$. A graph G is fully cycle extendable if every vertex in G lies on a cycle of length 3 and every non-hamiltonian cycle in G is extendable. A connected graph G is a split graph if the vertex set of G can be partitioned into a clique and a stable set. Dai et al. in [Discrete Mathematics, 345 (2022), 112826] conjectured that every $(r-1)$-connected $K_{1,r}$-free split graph is hamiltonian, and they proved this conjecture when $r = 4$ while Renjith and Sadagopan proved the case when $r = 3$. In this paper, we introduce a special type of alternating paths in the study of hamiltonian properties of split graphs and prove that a split graph G is hamiltonian if and only if G is fully cycle extendable. Consequently, for $r \in \{3, 4\}$, every r-connected $K_{1,r}$-free split graph is Hamilton-connect and every $(r-1)$-connected $K_{1,r}$-free split graph is fully cycle extendable. Joint work with Xia Liu, Mingquan Zhan and Hong-Jian Lai.

On tight (k, ℓ)-stable graphs

Zi-Xia Song, *University of Central Florida*
Saturday, April 20, 10:40 - 11:00, 220 Hodges Hall

For integers $k > \ell \geq 0$, a graph G is (k, ℓ)-stable if $\alpha(G - S) \geq \alpha(G) - \ell$ for every $S \subseteq V(G)$ with $|S| = k$. A recent result of Dong and Wu [SIAM J. Discrete Math. 36 (2022) 229–240] shows that every (k, ℓ)-stable graph G satisfies $\alpha(G) \leq \lfloor(|V(G)| - k + 1)/2\rfloor + \ell$. A (k, ℓ)-stable graph G is tight if

$\alpha(G) = \lfloor(|V(G)| - k + 1)/2\rfloor + \ell$; and q-tight for some integer $q \geq 0$ if $\alpha(G) = \lfloor(|V(G)| - k + 1)/2\rfloor + \ell - q$. In this talk, we first prove that for all $k \geq 24$, the only tight $(k, 0)$-stable graphs are K_{k+1} and K_{k+2}, answering a question of Dong and Luo [arXiv: 2401.16639]. We then prove that for all nonnegative integers k, ℓ, q with $k \geq 3\ell + 3$, every q-tight (k, ℓ)-stable graph has at most $k - 3\ell - 3 + 2^{3(\ell + 2q + 4)^2}$ vertices, answering a question of Dong and Luo in the negative.

This is joint work with Xiaonan Liu and Zhiyu Wang.

Cup Stacking in Graphs

Maya Tennant, *Virginia Commonwealth University*
Saturday, April 20, 2:20-2:40, 210 Hodges Hall

Here we introduce a game on graphs, called cup stacking, following a line of what can be considered as 0-, 1-, or 2-person games such as chip firing, percolation, graph burning, zero forcing, cops and robbers, graph pebbling, and graph pegging, among others. It can be more general, but the most basic scenario begins with a single cup on each vertex of a graph. For a vertex with k cups on it we can move all its cups to a vertex at distance k from it, provided the second vertex already has at least one cup on it. The object is to stack all cups onto some pre-described target vertex. We say that a graph is stackable if this can be accomplished for all possible target vertices. This game has been explored in some of these basic configurations as frog jumping on graphs.

In this paper we study cup stacking on many families of graphs, developing a characterization of stackability in graphs and using it to prove the stackability of complete graphs, paths, cycles, grids, the Petersen graph, many Kneser graphs, some trees, cubes of dimension up to 20, "somewhat balanced" complete t-partite graphs, and Hamiltonian diameter two graphs. Additionally we use the Gallai-Edmonds Structure Theorem, the Edmonds Blossom Algorithm, and the Hungarian algorithm to devise a polynomial algorithm to decide if a diameter two graph is stackable.

Our proof that cubes up to dimension 20 are stackable uses Kleitman's Symmetric Chain Decomposition and the new result of Merino, Mütze, and Namrata that all generalized Johnson graphs (excluding the Petersen graph) are Hamiltonian. We conjecture that all cubes and higher-dimensional grids are stackable, and leave the reader with several open problems, questions, and generalizations.

Joint work with Paul Fay and Glenn Hurlbert.

Removing long cycles in highly connected graphs with degree constraints

Lin Tian, *Middle Tennessee State University*
Saturday, April 20, 2:40 - 3:00, 214 Hodges Hall

In 1981, C. Thomassen proved that every $(k + 3)$-connected graph G contains a cycle C such that $\kappa(G - V(C)) \geq k$. Further, he proved that every $(s + t - 1)$-connected graph G of minimum degree at least $g(n(s), n(t))$, its vertex set can be decomposed into two sets S and T such that the subgraphs $G[S]$ and $G[T]$ have connectivity at least s and t, respectively. In this paper, we prove that every $(k + 1)$-connected graph with minimum degree at least $\frac{3k}{2} + 2m - 3$ contains a cycle of length at least m whose removal remains k-connected. Our proof leads to a polynomial algorithm for finding a such long cycle in $(k + 1)$-connected graphs.

Some problems on r-hued colorings

Murong Xu, *University of Scranton*
Saturday, April 20, 10:20-10:40, 210 Hodges Hall

Let k, r, d be positive integers. A (k, r)-coloring of a graph G is a proper k-vertex coloring of G such that the neighbors of each vertex of degree d will receive at least $\min(d, r)$ different colors. The r-hued chromatic number is the smallest integer k for which a graph G has a (k, r)-coloring. In this talk, I will introduce the recent developments on the studies related to this r-hued coloring and discuss some of the open problems on it.

Joint work with Ye Chen, Suohai Fan, and Hong-Jian Lai.

Cycle Covers of Graphs

Dong Ye, *Middle Tennessee State University*
Saturday, April 20, 11:00 - 12:00, 202 Hodges Hall

Let G be a graph. A cycle cover of a graph G is a family of cycles which contain every edge of G. A cycle double cover is a family of cycles such that every edge is contained by exactly two cycles in the family. The well-known cycle double cover conjecture, due to Seymour and Szekeres, asserts that every bridgeless graph has a cycle double cover. In this talk, we will focus on some old and new results on cycle covers of graphs.

On the $(1^2, 2^4)$-packing edge-coloring of subcubic graphs

Gexin Yu, *College of William and Mary*
Saturday, April 20, 2:00 - 2:20, 210 Hodges Hall

An induced matching in a graph G is a matching such that its end vertices also induce a matching. A $(1^\ell, 2^k)$-packing edge-coloring of a graph G is a partition of its edge set into disjoint unions of ℓ matchings and k induced matchings. Gastineau and Togni (2019), as well as Hocquard, Lajou, and Lužar (2022), have conjectured that every subcubic graph is $(1^2, 2^4)$-packing edge-colorable. In this paper, we confirm that their conjecture is true (for connected subcubic graphs with more than 70 vertices). Our result is sharp due to the existence of subcubic graphs that are not $(1^2, 2^3)$-packing edge-colorable. This is joint work with Xujun Liu.

Pancyclicity of line graphs with diameter two

Mingquan Zhan, *Millersville University of Pennsylvania*
Saturday, April 20, 10:00-10:20, 214 Hodges Hall

A graph G is pancyclic if it contains cycles of all possible lengths. In [J. Graph Theory, 12 (1988) 413-420] Veldman showed that the line graph $L(G)$ of a connected graph G with diameter at most 2 is hamiltonian. In this paper, we continue studying the line graph $L(G)$ of a connected graph G with $|E(G)| \geq 3$ and diameter at most 2 and prove that $L(G)$ is pancyclic if and only if G is not a cycle of length 4 or 5, and G is not the Petersen graph. This is a joint work with Drs. Xiaoling Ma, Lan Lei, and Hong-Jian Lai.

Efficient Dominating Sets

Ju Zhou, *Kutztown University of Pennsylvania*
Saturday, April 20, 10:40 - 11:00, 214 Hodges Hall

Given a finite simple graph G, a set $D \subset V(G)$ is said to be a dominating set if for all $v \in V(G)$, either $v \in D$ or v is adjacent to some vertex in D. For a graph G, the domination number is the size of the smallest possible dominating set. If D is a dominating set of G such that $|D|$ equals the domination number of G, then D is called a minimum dominating set. Furthermore, D is called a unique minimum dominating set if there is precisely one minimum dominating set D. Given a graph G, a dominating set D is an independent dominating set if none of the vertices in D are adjacent. If each vertex not in D is adjacent to precisely one element in D, then D is a perfect dominating set. If D is both independent and perfect, it is an efficient dominating set. In this paper, the authors discussed some properties of efficient dominating sets and then investigated several family graphs about whether they have efficient dominating sets.

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